



Munich Personal RePEc Archive

# Estimating risk aversion from ascending and sealed-bid auctions: the case of timber auction data

Jingfeng Lu and Isabelle Perrigne

November 2006

Online at <http://mpra.ub.uni-muenchen.de/948/>

MPRA Paper No. 948, posted 28. November 2006

**Estimating Risk Aversion  
from Ascending and Sealed-Bid Auctions:  
The Case of Timber Auction Data**

Jingfeng Lu

National University of Singapore

Isabelle Perrigne

Pennsylvania State University

September 2006

\* We thank Phil Haile for providing the data analyzed in this paper, Quang Vuong for thoughtful discussions and Michael Visser for comments. The second author is grateful to the National Science Foundation for research support through grant SES-0452154.

Correspondence to: I. Perrigne, Department of Economics, Pennsylvania State University, University Park, PA 16802. e-mail address : [perrigne@psu.edu](mailto:perrigne@psu.edu).

## **Abstract**

Estimating bidders' risk aversion in auctions is a challenging problem because of identification issues. This paper takes advantage of bidding data from two auction designs to identify nonparametrically the bidders' utility function within a private value framework. In particular, ascending auction data allow us to recover the latent distribution of private values, while first-price sealed-bid auction data allow us to recover the bidders' utility function. This leads to a nonparametric estimator. An application to the US Forest Service timber auctions is proposed. Estimated utility functions display concavity, which can be partly captured by constant relative risk aversion.

Key words: Risk Aversion, Nonparametric Identification, Nonparametric and Semiparametric Estimation, Timber Auctions.

# Estimating Risk Aversion from Ascending and Sealed-Bid Auctions: The Case of Timber Auction Data

J. Lu and I. Perrigne

## 1 Introduction

The concept of risk aversion is at the core of economic agents' decisions under uncertainty. Since the formalization of risk aversion by Pratt (1964), a rich theoretical literature has developed models to explain how agents behave in such situations. In auctions, bidders face many uncertainties related to the auction game while auctioned objects may represent an important value relative to their assets. The auction model and the optimal mechanism design with risk averse bidders have been studied by Maskin and Riley (1984) and Matthews (1987). In particular, first-price sealed-bid auctions dominate ascending auctions within the private value paradigm, while the optimal auction design involves some transfers among bidders. Within the private value paradigm, risk averse bidders tend to shade less their private values relative to the risk neutral case leading to some overbidding. This provides the intuition of the dominance of the first-price sealed-bid mechanism over the ascending one as announcing his private value is still a dominant strategy in the latter. More recently, Eso and White (2004) have introduced the concept of precautionary bidding when bidders face uncertainties about the ex post realizations of their values. In parallel to this theoretical literature, experimental data have suggested

that bidders tend to bid above the Bayesian Nash equilibrium, which can be explained by bidders' risk aversion. Such an approach has been adopted by Cox, Smith and Walker (1988) and Bajari and Hortacsu (2005) among others, while Goere, Holt and Palfrey (2002) also found potential bidders' risk aversion in a quantile response equilibrium.<sup>1</sup> On empirical grounds, few empirical studies have assessed bidders' risk aversion. Athey and Levin (2001) study bidding on species in timber auctions with ex post payments based on actual harvested values. Their empirical analysis suggests that bidding behavior is consistent with risk aversion as bidders tend to diversify risk across species. Potential bidders' risk aversion has also been found in timber auctions by Baldwin (1995) within a reduced form approach and by Perrigne (2003) within a structural approach.

Previous experimental and empirical studies have adopted a parametric approach, while specifying a known form of risk aversion for the bidders' utility function such as constant relative risk aversion (CRRA) or constant absolute risk aversion (CARA). This also includes Campo, Guerre, Perrigne and Vuong (2006) relying on a semiparametric approach. Both families of utility functions provide simple functional forms, while a CRRA specification encompasses the case of risk neutrality.<sup>2</sup> As a matter of fact, little is known on the shape of bidders' utility function. The choice of a family of utility functions may affect the estimated results and have misleading implications on bidders' behavior. Moreover, there is no general agreement in the theoretical literature on which concept of risk aversion is the most appropriate to explain observed phenomena such as overbidding in auctions. See Gollier (2001) for a survey on risk aversion. A nonparametric approach that leaves unspecified such a utility function will shed some lights on agents' economic behavior facing risk. A parametric specification of the utility function has been justified

---

<sup>1</sup>Relying on recent structural econometric methods, Bajari and Hortacsu (2005) estimate several models to explain experimental data and find that risk aversion provides the best fit.

<sup>2</sup>In the case of auctions, the Bayesian Nash equilibrium strategy has a closed form solution for a CRRA specification of the bidders' utility function within the independent private value paradigm. This is not the case for a CARA specification. This can explain the popularity of the CRRA utility function in the auction literature.

so far by the difficulties of identifying such a function in auction models. Campo, Guerre, Perrigne and Vuong (2006) show that the auction model with risk averse bidders is identified semiparametrically within the private value paradigm. A parametric specification of the utility function is a necessary identifying restriction in addition to a conditional quantile restriction on the bidders' private value distribution, which is left unspecified. Campo (2005) obtains a semiparametric identification result of the auction model with heterogeneous bidders, while maintaining a parametric specification for the bidders' utility functions. More recently, Guerre, Perrigne and Vuong (2006) exploit some exclusion restrictions such as an exogenous bidders' participation leading to a latent distribution of private values independent of the number of bidders to identify nonparametrically the bidders' utility function. They extend their results to an endogenous bidders' participation with the availability of instruments that do not affect the bidders' private value distribution. Their identification result leads to the construction of an infinite series of differences in quantiles. The resulting estimator can be quite burdensome to implement.

In this paper, we choose a fully nonparametric approach while exploiting additional bidding data. This approach is intuitive and the resulting estimator is straightforward to implement. The US Forest Service (USFS) uses both ascending and sealed-bid auctions to sell its standing timber. To our knowledge, two other papers by Hansen (1985) and Athey, Levin and Seira (2004) exploit the two auction design data for different purposes. The former attempts to test the revenue equivalence theorem, while the latter studies the entry and bidding patterns in both ascending and sealed-bid auctions with asymmetric bidders to explain the choice of the auction mechanism. We show that the data from these two auction designs can be used to identify nonparametrically the bidders' utility function. The intuition is as follows. Since the bidding strategy in ascending auctions is not affected by risk aversion, the observed winning bids in ascending auctions identify the underlying private value distribution using the distribution of order statistics as shown by Athey and Haile (2002). Following Guerre, Perrigne and Vuong (2000), we use the monotonicity of the bidding strategy relating the bid to the private value to rewrite the differential

equation defining the equilibrium strategy as a function of the bid distribution and density. The derived equation allows us to identify nonparametrically the utility function when the private value distribution is known. We then derive a multistep nonparametric estimation procedure. We can then assess whether the widely used CRRA or CARA specifications adjust the recovered utility function while using a semiparametric model. On empirical grounds, we need to pay special attention to the data. In particular, the same bidders need to participate to both auctions. If not the case, we may face different underlying private value distributions, which would invalidate our approach. Moreover, data show that ascending auctions tend to be chosen over first-price sealed-bid auctions for parcels with a large timber volume leading to some sample selection. The empirical results show that the model is not rejected by the data. Estimated utility functions are increasing and display some concavity. Though we cannot find a perfect adjustment, a CRRA specification better captures the bidders' behavior than a CARA specification.

The paper is organized as follows. A second section is devoted to the auction model, its nonparametric identification and estimation. A third section introduces the bidding data and addresses the issues raised previously, while a fourth section presents the empirical results. A fifth section concludes.

## **2 Auction Models, Identification and Estimation**

This section briefly presents the ascending and first-price sealed-bid auction models with risk averse bidders within a private value framework. The identification of the model structure is addressed using bidding data from these two auction designs. The bidders' utility function is nonparametrically identified. This naturally leads to a nonparametric estimator.

## 2.1 The Ascending and First-Price Sealed-Bid Auctions

A single and indivisible object is sold through an auction to a number  $I$  of bidders. We consider the private value paradigm where every bidder has a private value  $v_i$  for the auctioned object. The private values  $v_i, i = 1, \dots, I$  are drawn independently from a distribution  $F(\cdot)$ , which is known to all bidders. This distribution is defined on a compact support  $[\underline{v}, \bar{v}]$  with a density  $f(\cdot)$ . We assume that every bidder is potentially risk averse with a von Neuman Morgensten utility function  $U(\cdot)$  satisfying  $U'(\cdot) > 0$ ,  $U''(\cdot) \leq 0$  and  $U(0) = 0$ . The bidders are symmetric in the sense that they share the same private value distribution  $F(\cdot)$  and the same utility function  $U(\cdot)$ .<sup>3</sup> Hereafter,  $b_i$  denotes bidder's  $i$  bid. For simplicity, we consider an auction with a nonbinding reserve price. The ascending auction model takes a rather simple form. Whatever the level and the shape of risk aversion, it is a dominant strategy for each bidder to bid his private value  $v_i$ , namely  $b_i = v_i$  since he will pay the second-highest bid when he wins the auction. Thus risk aversion does not affect bidding behavior.

In the case of a first-price sealed-bid auction, the outcome is different as bidders tend to overbid relative to the risk neutral case. In a first-price sealed-bid auction, bidder  $i$  maximizes his expected gain from the auction, namely  $U(v_i - b_i)\Pr(b_i \geq b_j, j \neq i)$ , where  $v_i - b_i$  expresses the monetary gain. Let  $s(\cdot, U, F, I)$  be the strictly increasing symmetric Bayesian Nash equilibrium strategy with  $s^{-1}(\cdot)$  denoting its inverse. Because of independence, the probability of winning the auction is then equal to  $F^{I-1}(s^{-1}(\cdot))$ . Maximizing the expected gain for bidder  $i$  with respect to his bid  $b_i$  and imposing  $b_i = s(v_i)$  give the following differential equation

$$s'(v_i) = (I - 1) \frac{f(v_i)}{F(v_i)} \lambda(v_i - b_i), \quad (1)$$

---

<sup>3</sup>We can extend our model while incorporating a common wealth  $w \geq 0$ . Considering individual wealth  $w_i$  will lead to an asymmetric game if the  $w_i$ s are ex ante known to all bidders or to a multisignal game if the  $w_i$ s are private information. The first case would require additional data on bidders' wealth, while the second case would lead to a complex model beyond the scope of this paper.



for  $v_i \in [\underline{v}, \bar{v}]$ , where  $\lambda(\cdot) = U(\cdot)/U'(\cdot)$ .<sup>4</sup> The function  $\lambda(\cdot)$  is strictly increasing on its compact support with a lower bound at zero since  $\lambda(0) = 0$  following  $\underline{v} = s(\underline{v})$ . The upper bound is noted  $\overline{v - b}$ , which corresponds to the maximum value for the bidder's gain  $v - b = v - s(v)$ . Note that if  $v - s(v)$  is increasing in  $v$ , this upper bound becomes  $\bar{v} - s(\bar{v})$ . The above differential equation does not have a closed form solution for  $s(\cdot)$  under a general utility specification. When  $U(\cdot)$  belongs to the CRRA utility family, an analytical expression for  $s(\cdot)$  can be derived. Hereafter, the pair  $[U, F]$  is defined as the structure of the game.

## 2.2 Nonparametric Identification of Bidders' Utility Function

Before addressing the problem of identification, it is useful to define the observables. We assume that two auction designs are used to sell similar objects and that bidders participate to both auctions. We will later introduce a vector of characteristics to entertain the case of different products. The  $L_1$  ascending auctions indexed by  $\ell$  provide the winning bids  $b_{w\ell}, \ell = 1, \dots, L_1$ , while the  $L_2$  first-price sealed-bid auctions provide the sequence of bids  $b_{i\ell}, i = 1, \dots, I, \ell = 1, \dots, L_2$ . Following Athey and Haile (2002, Theorem 1), noting that the observed winning bid in an ascending auction is the second highest private value, the private value distribution can be identified using order statistics.<sup>5</sup> For instance, when  $I = 2$ , the observed winning bids  $b_{w\ell}, \ell = 1, \dots, L_1$  are the private values of the losers. When  $I = 3$ , the observed winning bids are the private values for the second-highest bidders or the second-order statistics. Using the distribution of the  $n$ th order statistics, the distribution of private values can be recovered using

$$F^{n,I}(v) = \frac{I!}{(n-1)!} \int_0^{F(v)} t^{n-1} (1-t)^{I-n} dt, \quad (2)$$

---

<sup>4</sup>As usual, this differential equation is subject to a singularity problem at the lower bound. As shown by Maskin and Riley (1984), the boundary condition is  $U(\underline{v} - s(\underline{v})) = 0$ , i.e.  $s(\underline{v}) = \underline{v}$  since  $U(0) = 0$ .

<sup>5</sup>This result implicitly assumes that the number of bidders  $I$  is known, while only the winner's bid is observed. The USFS timber auction data provides information on the number of bidders.

with  $n = I - 1$  since the highest bid is equal to the  $(I - 1)$ th order statistics. For  $n = 1$  and  $I = 2$ , this gives  $F^{1,2}(v) = 2F(v) - F^2(v)$ , from which we can find a unique solution for  $F(\cdot)$  satisfying the properties of a cdf. More generally, we can use  $F^{I-1,I}(v) = IF(v)^{I-1} - (I - 1)F(v)^I$ , from which we can recover uniquely  $F(v)$ . Thus the ascending auction data allow us to recover the latent distribution of private values. The ascending auction data do not allow us, however, to recover any information on the bidders' utility function and a fortiori on their risk aversion. Note that, if we consider a more general model with affiliated private values, the observation of the winning bid will not be sufficient to recover the latent distribution of bidders' private values as shown by Athey and Haile (2002). Independence of private values is a key assumption here.

Using this information, we can now address the problem of identification of the function  $U(\cdot)$  using first-price sealed-bid auction data. Using a similar argument as in Guerre, Perrigne and Vuong (2000), we use the distribution of equilibrium bids  $G(\cdot)$  to reformulate the differential equation (1). For every  $b \in [\underline{b}, \bar{b}] = [\underline{v}, s(\bar{v})]$ , we have  $G(b) = F(s^{-1}(b)) = F(v)$  with density  $g(b) = f(v)/s'(v)$ , where  $v = s^{-1}(b)$ . After elementary algebra and since  $\lambda(\cdot)$  is strictly increasing, we obtain

$$v_{i\ell} = \xi(b_{i\ell}) = b_{i\ell} + \lambda^{-1} \left( \frac{1}{I-1} \frac{G(b_{i\ell})}{g(b_{i\ell})} \right), \quad (3)$$

$i = 1, \dots, I, \ell = 1, \dots, L_2$ , where  $\lambda^{-1}(\cdot)$  denotes the inverse of  $\lambda(\cdot)$  and  $\xi(\cdot)$  is the inverse equilibrium strategy. Since the private value distribution and the bid distribution are known, the function  $\lambda(\cdot)$  is identified. The next proposition shows that  $U(\cdot)$  is nonparametrically identified.

**Proposition 1:** *Let  $I \geq 2$ . Assuming  $F(\cdot)$  is known,  $U(\cdot)$  is nonparametrically identified. Thus, any pair  $[U, F]$  is identified when bidding data from ascending and first-price sealed-bid designs are combined.*

**Proof:** Let  $v(\alpha)$  and  $b(\alpha)$  be the  $\alpha$ -quantiles of the distributions  $F(\cdot)$  and  $G(\cdot)$ , respec-

tively, with  $\alpha \in [0, 1]$ , namely  $v(\alpha) = F^{-1}(\alpha)$  and  $b(\alpha) = G^{-1}(\alpha)$ . Using (3) gives

$$\lambda(F^{-1}(\alpha) - G^{-1}(\alpha)) = \frac{1}{I-1} \frac{\alpha}{g(G^{-1}(\alpha))}.$$

Since  $I$ ,  $F(\cdot)$ ,  $G(\cdot)$  and  $g(\cdot)$  are known,  $\lambda(\cdot)$  is nonparametrically identified on its support  $[0, \overline{v - b}]$ . In particular, at  $\alpha = 0$ ,  $F^{-1}(0) = \underline{v} = \underline{b} = G^{-1}(0)$ , while  $\overline{v - b} = \max_{\alpha} F^{-1}(\alpha) - G^{-1}(\alpha)$ . Moreover, the function  $U(\cdot)$  can be identified up to scale since

$$U(x) = A \exp \int_{\underline{v-b}}^x \frac{1}{\lambda(t)} dt,$$

for  $x \in [0, \overline{v - b}]$ , for any value  $A > 0$ .  $\square$

Proposition 1 tells us that, since  $F(\cdot)$  is identified from the ascending auction bid data, the first-price sealed-bid data allow us to identify nonparametrically the utility function  $U(\cdot)$ .<sup>6</sup> Note that  $U(\cdot)$  can be identified up to scale. Since the scale is irrelevant, we can impose without loss of generality a normalization such as  $U(1) = 1$ .<sup>7</sup> Note that our results can be generalized to the case of an announced reserve price. In this case, only bidders who have a private value above the reserve price  $p_0$  will bid. Thus, the number of potential bidders  $I$  is unknown. Using an argument as in Guerre, Perrigne and Vuong (2000),  $I$  is identified. Using an estimate for the number of bidders  $I$ , the ascending auction data will allow us to recover the truncated distribution of private values. Equation (3) would need to be adapted since the observed bids in the first-price sealed-bid auction are also distributed following a truncated distribution. It can be easily shown that the argument of the inverse of  $\lambda(\cdot)$  in (3) becomes  $[1/(I-1)] \times \{[G^*(b_i)/g^*(b_i)] + F(p_0)/[(1-F(p_0))g^*(b_i)]\}$ , where  $G^*(\cdot)$  and  $g^*(\cdot)$  denote the truncated bid distribution and density, respectively. Relying on Guerre, Perrigne and Vuong (2000, Theorem 4), Proposition 1 extends.

Bidders' asymmetry can be easily entertained. We assume that bidders' identity is known and that bidders' asymmetry is known ex ante to all bidders. Asymmetry may

---

<sup>6</sup>There are some natural restrictions on the set of identifiable  $U(\cdot)$  functions and  $F(\cdot)$  distributions. See Definitions 1 and 2 in Campo, Guerre, Perrigne and Vuong (2006).

<sup>7</sup>If we consider a model with a common wealth,  $\lambda(\cdot)$  becomes  $[U(w + \cdot) - U(w)]/U'(w + \cdot)$ . We cannot identify nonparametrically  $[U, F, w]$ . A parameterization of  $U(\cdot)$  becomes necessary to identify the model.

arise from different private value distributions and/or heterogeneity in preferences. While considering asymmetry in private values, let  $v_i$  denote the bidder's  $i$  private value distributed as  $F_i(\cdot)$  on  $[\underline{v}, \bar{v}_i]$ . It is still a dominant strategy in an ascending auction to bid his private value, i.e.  $b_i = v_i$ . Athey and Haile (2005) and Brendstrup and Paarsch (2006) show that the distributions  $[F_1, \dots, F_I]$  are identified using the distributions of order statistics. Bidding in first-price sealed-bid auctions takes a more complex form leading to a possibly inefficient allocation. In particular, following Campo, Perrigne and Vuong (2003) and Flambard and Perrigne (2006), (3) becomes

$$v_{i\ell} = b_{i\ell} + \lambda^{-1} \left( \frac{1}{H_i(b_{i\ell})} \right), \quad (4)$$

$i = 1, \dots, I, \ell = 1, \dots, L_2$ , where  $H_i(\cdot) = \sum_{k \neq i} g_k(\cdot)/G_k(\cdot)$ . Proposition 1 can be easily extended using the  $v_i(\alpha)$  and  $b_i(\alpha)$  quantiles of the recovered private value distributions  $F_i(\cdot)$  and observed bid distributions  $G_i(\cdot)$ , respectively. Asymmetry in bidders' preferences can be entertained in a similar manner. The structure of the game becomes  $[U_1, \dots, U_I, F]$ . Bidding in ascending auctions is not affected, while the inverse equilibrium strategies can be written as in (4) with  $\lambda_i(\cdot)$  instead of  $\lambda(\cdot)$ . For every bidder  $i$ , the  $v(\alpha)$  and  $b_i(\alpha)$  quantiles can be used to identify  $\lambda_i(\cdot)$ .<sup>8</sup> Asymmetry in both private values and preferences can also be entertained despite its complexity. The structure of the game becomes  $[U_1, \dots, U_I, F_1, \dots, F_I]$ . Similar arguments can be used to extend Proposition 1.

## 2.3 Nonparametric Estimation

We now consider the case where auctioned objects are heterogeneous. Namely, each auctioned object can be characterized by a  $d$ -dimensional vector  $Z_\ell \in \mathcal{Z} \subset \mathbb{R}^d$ . We assume that the analyst shares the same information as bidders, i.e. there is no unobserved heterogeneity. The number of bidders may also differ across auctions and

---

<sup>8</sup>As noted in Campo, Guerre, Perrigne and Vuong (2006, Section 7.3), such a model leads to some compatibility conditions under the form  $b_i(\alpha) + \lambda_i^{-1}(1/H_i(b_i(\alpha))) = b_j(\alpha) + \lambda_j^{-1}(1/H_j(b_j(\alpha)))$  for any  $\alpha \in [0, 1]$  and any pair of bidders  $(i, j), i \neq j$ .

will be indexed by  $\ell$ , which gives  $I_\ell$ . The observations in the ascending auctions are  $\{b_{w\ell}, Z_\ell, I_\ell, \ell = 1, \dots, L_1\}$ , while the observations in the first-price sealed-bid auctions are  $\{b_{i\ell}, i = 1, \dots, I_\ell, Z_\ell, I_\ell, \ell = 1, \dots, L_2\}$ . While introducing this heterogeneity, we can make some assumptions on the structure  $[U, F]$  of the game. In particular, for every  $\ell$ , the  $v_{i\ell}, i = 1, \dots, I_\ell$  are independently and identically distributed conditionally upon  $(Z_\ell, I_\ell)$  as  $F(\cdot|Z_\ell, I_\ell)$ . Thus we do not exclude an endogenous number of bidders, which may affect the distribution of bidders' private values. The comparison of the estimated conditional private value distributions for different values of the number of bidders allows us in principle to test for the exogeneity of the number of bidders. Regarding the function  $U(\cdot)$ , our identification result allows us to entertain a general case in which  $U(\cdot)$  may be considered as conditional on  $I_\ell$ , though a natural assumption would be to restrict  $U(\cdot)$  to be independent of the number of bidders. The comparison of the estimated function  $U(\cdot)$  for different values of  $I$  can be used to test such a restriction.

The estimation procedure is in several steps. In a first step, the ascending auction data are used to estimate nonparametrically the conditional distribution of private values. In a second step, using the estimated conditional private value distribution, the first-price sealed-bid data are used to estimate nonparametrically the function  $\lambda(\cdot)$  from which the function  $U(\cdot)$  can be estimated. A third step may consist in using the estimated function  $\lambda(\cdot)$  to assess the adjustment with a parametric function of the form  $\lambda(\cdot; \theta)$  derived from a CRRA or CARA model.

Since the identification result for the latent private value distribution relies on order statistics, the first step of the estimation procedure needs to be conducted for every possible value  $I$ . We denote by  $L_{1I}$  and  $L_{2I}$  the number of ascending and sealed-bid auctions corresponding to the number of bidders  $I$ , respectively. In particular,  $F(v|z, I)$  is defined as the solution of

$$F^{(I-1)}(v|z, I) = IF^{I-1}(v|z, I) - (I-1)F^I(v|z, I),$$

for any value  $(z, I)$ , where  $F^{(I-1)}(\cdot|z, I)$  is the conditional distribution of the  $(I-1)$ th

order statistic of observed winning bids. In particular, for  $I = 2$ , we have  $F(v|z, I) = 1 - \sqrt{1 - F^{(1)}(v|z, I)}$ . For  $I \geq 3$ , there is no explicit solution to the above equation and numerical algorithms need to be used to solve the equation. The distribution of the order statistic  $F^{(I-1)}(v|z, I)$  can be estimated using a standard nonparametric kernel estimator. In particular,

$$\hat{F}^{(I-1)}(v|z, I) = \frac{\frac{1}{L_{1I}h_F^d} \sum_{\ell=1}^{L_{1I}} \mathbb{I}(b_{w\ell} \leq v) K\left(\frac{z-Z_\ell}{h_F}\right)}{\frac{1}{L_{1I}h_F^d} \sum_{\ell=1}^{L_{1I}} K\left(\frac{z-Z_\ell}{h_F}\right)}, \quad (5)$$

for an arbitrary value  $(v, z, I)$ , where  $K(\cdot)$  is a kernel function and  $h_F$  a vanishing bandwidth. The kernel function can be chosen satisfying standard assumptions. In the following section, we will consider a triweight kernel of the form  $K(u) = (35/32)(1-u^2)^3 \mathbb{I}(|u| \leq 1)$ .<sup>9</sup> The bandwidth needs particular attention. If we assume that  $F(\cdot|\cdot, \cdot)$  is  $R+1$  continuously differentiable, the bandwidth needs to be chosen with the following vanishing rate,  $h_F \propto L_{1I}^{-1/(2R+d+2)}$ . Note that this bandwidth corresponds to the optimal uniform consistency rate as defined by Stone (1982), i.e.  $L_{1I}^{(R+1)/(2R+d+2)}$ . Thus, the conditional density  $f(\cdot|\cdot)$  can be estimated at the optimal rate  $L_{1I}^{R/(2R+d+1)}$ . We attain the optimal consistency rate since we estimate the latent private value distribution directly from observables and not from recovered (estimated) private values as in Guerre, Perrigne and Vuong (2000), where the consistency rate is lower. As discussed previously, comparing the estimated distributions  $\hat{F}(\cdot|\cdot, \cdot)$  for different values of the number of bidders such as  $I = 2, 3, 4, \dots$ , if data permit, would allow us in principle to test their equality across  $I$ . If such distributions are equal, we could then consider that the number of bidders is exogenous.

The second step makes use of the first-price sealed-bid auction data. The idea is to

---

<sup>9</sup>The triweight kernel is chosen for convenience. This kernel is of order 2, though Proposition 2 would require to use a kernel of order 4 or above. Because a higher order kernel involves negative estimated densities and the choice of the kernel function does not have much effect in practice, we have preferred to use a standard kernel function. Moreover, the multivariate triweight kernel is the product of univariate triweight kernels.

recover the function  $\lambda(\cdot)$  using (3). To do so, we need first to estimate the conditional bid distribution and density. Since bids depend on the number of bidders through the equilibrium strategy, it is expected that the bid distribution also depends on the number of bidders. Thus, we consider the conditional distribution  $G(b|z, I)$  and density  $g(b|z, I)$  for any arbitrary value  $(b, z, I)$ . Using kernel estimators, we obtain

$$\hat{G}(b|z, I) = \frac{\frac{1}{IL_{2I}h_G^d} \sum_{\ell=1}^{L_{2I}} \sum_{i=1}^I \mathbb{I}(b_{i\ell} \leq b) K\left(\frac{z-Z_\ell}{h_G}\right)}{\frac{1}{L_{2I}h_G^d} \sum_{\ell=1}^{L_{2I}} K\left(\frac{z-Z_\ell}{h_G}\right)} \quad (6)$$

$$\hat{g}(b|z, I) = \frac{\frac{1}{IL_{2I}h_g^{d+1}} \sum_{\ell=1}^{L_{2I}} \sum_{i=1}^I K\left(\frac{b-b_{i\ell}}{h_g}\right) K\left(\frac{z-Z_\ell}{h_g}\right)}{\frac{1}{L_{2I}h_g^d} \sum_{\ell=1}^{L_{2I}} K\left(\frac{z-Z_\ell}{h_g}\right)}, \quad (7)$$

where  $K(\cdot)$  is a kernel function and  $h_G$  and  $h_g$  are two vanishing bandwidths. As usual, special attention should be given to the bandwidths. As shown by Campo, Guerre, Perrigne and Vuong (2006), the smoothness of the underlying distribution implies some smoothness conditions on the bid distribution. We maintain the assumptions of Campo, Guerre, Perrigne and Vuong (2006), namely  $F(\cdot|\cdot)$  and  $\lambda(\cdot)$  are both  $R+1$  continuously differentiable leading to a  $R+1$  continuously differentiable equilibrium strategy  $s(\cdot)$ . Consequently,  $G(\cdot|\cdot)$  is  $R+1$  continuously differentiable and the model implies that  $g(\cdot|\cdot)$  is also  $R+1$  continuously differentiable.<sup>10</sup> The bandwidths need to be chosen accordingly, namely  $h_G \propto (IL_{2I})^{-1/(2R+d+2)}$  and  $h_g \propto (IL_{2I})^{-1/(2R+d+3)}$  leading to optimal consistency rates for estimating  $G(\cdot|\cdot, I)$  and  $g(\cdot|\cdot, I)$ , i.e.  $(IL_{2I})^{(R+1)/(2R+d+2)}$  and  $(IL_{2I})^{(R+1)/(2R+d+3)}$ , respectively.

To recover the function  $\lambda(\cdot)$ , we use the quantiles of the distributions  $F(\cdot|\cdot, I)$  and  $G(\cdot|\cdot, I)$  and exploit the relationship  $G(b_{i\ell}|Z_\ell, I_\ell) = F(v_{i\ell}|Z_\ell, I_\ell)$ . The idea is as follows. To any observed bid  $b_{i\ell}, i = 1, \dots, I_\ell, \ell = 1, \dots, L_2$ , it corresponds an  $\alpha$ -quantile  $b(\alpha, Z_\ell, I_\ell)$  such that  $b(\alpha, Z_\ell, I_\ell) = b_{i\ell}$  and  $G(b(\alpha, Z_\ell, I_\ell)|Z_\ell, I_\ell) = \alpha$ . From the private value distribution, it corresponds an  $\alpha$ -quantile  $v(\alpha, Z_\ell, I_\ell)$ . Since  $G(b_{i\ell}|Z_\ell, I_\ell) = F(v_{i\ell}|Z_\ell, I_\ell)$ , the

---

<sup>10</sup>This can be easily shown using (3), which gives  $g(b|z, I) = G(b|z, I)/[(I-1)\lambda(\xi(b)-b)]$ . Note that the estimation is performed for every value of  $I$  since  $I$  takes discrete values.

corresponding private value  $v_{i\ell}$  is equal to this quantile, i.e.  $v_{i\ell} = F^{-1}(G(b_{i\ell}|Z_\ell, I_\ell)|Z_\ell, I_\ell)$ . We can use the results of Bhattacharya and Gangopadhyay (1990) and Chaudhuri (1991) to obtain the estimated quantiles  $\hat{b}(\alpha, Z_\ell, I_\ell)$  and  $\hat{v}(\alpha, Z_\ell, I_\ell)$ . The estimated private value is obtained as  $\hat{v}_{i\ell} = \hat{F}^{-1}(\hat{G}(b_{i\ell}|Z_\ell, I_\ell)|Z_\ell, I_\ell)$  using a kernel estimator for the bid distribution as defined in (6). It remains to estimate  $\lambda(\cdot)$ . Using (3), we obtain

$$\frac{1}{I_\ell - 1} \frac{G(b_{i\ell}|Z_\ell, I_\ell)}{g(b_{i\ell}|Z_\ell, I_\ell)} = \lambda(v_{i\ell} - b_{i\ell}),$$

$i = 1, \dots, I_\ell, \ell = 1, \dots, L_2$ . To obtain an estimate for  $\lambda(\cdot)$ , we need to replace the unknown distribution, density and value by their estimates, namely  $\hat{G}(b_{i\ell}|Z_\ell, I_\ell)$ ,  $\hat{g}(b_{i\ell}|Z_\ell, I_\ell)$  and  $\hat{v}_{i\ell} = \hat{F}^{-1}(\hat{G}(b_{i\ell}|Z_\ell, I_\ell)|Z_\ell, I_\ell)$ . At a given  $I$ , this will give  $IL_{2I}$  pairs  $(\hat{v}_{i\ell} - b_{i\ell}, \hat{G}(b_{i\ell}|Z_\ell, I_\ell)/((I_\ell - 1)\hat{g}(b_{i\ell}|Z_\ell, I_\ell)))$ , which will trace out the function  $\lambda(\cdot)$ . The function  $\lambda(\cdot)$  can then be estimated by smoothing the scatter plot of these pairs. This procedure, which is very intuitive, is relatively easy to implement. Given the above consistency rates for estimating  $G(\cdot|\cdot)$ ,  $g(\cdot|\cdot)$  and  $F(\cdot|\cdot)$ ,  $\lambda(\cdot)$  is estimated at the rate  $(IL_{2I})^{(R+1)/(2R+d+3)}$  assuming that  $(L_{1I}/L_{2I}) \rightarrow a$ , where  $0 < a < \infty$ .

The previous estimated distributions and functions can be used to test the validity of the model. Based on (3), a first prediction of the model is that the private value must be larger than the corresponding bid since  $\lambda(\cdot)$  is an increasing function defined on  $[0, \overline{v - b}]$  and  $\lambda(0) = 0$ . Such a prediction can be tested using the estimated conditional distributions  $\hat{F}(\cdot|z, I)$  and  $\hat{G}(\cdot|z, I)$ . As a matter of fact, the bid distribution defined on  $[\underline{v}(z, I), s(\overline{v}(z, I))]$  should stochastically dominate the private value distribution defined on  $[\underline{v}(z, I), \overline{v}(z, I)]$ . If it is not the case, this may suggest that the observed bids in the first-price sealed-bid auctions are the outcomes of another private value distribution than the one recovered from the winning bids in ascending auctions. This may arise if the pool of bidders is quite different across the two auction designs. Alternatively, we know from the model that the function  $\lambda(\cdot)$  is continuous and strictly increasing. This implies that, for an arbitrary pair  $(z, I)$ , a unique value  $G(b|z, I)/((I - 1)g(b|z, I))$  should correspond to the gain  $v - b$ . Several reasons can be invoked if such a mapping does not hold.



As suggested before, it may arise from the fact that the observed bids in the first-price sealed-bid auctions are not generated by the private value distribution estimated from the winning bids in the ascending auctions because of a different pool of bidders. It may also arise when the model under consideration is inappropriate to explain the bids. Such tests do not pretend to test fully the validity of the model but they represent some steps in that direction in the sense that rejection of such tests would clearly indicate a mismatch of the model to explain the bidding data.

The scatter plot of the function  $\hat{\lambda}(\cdot)$  can also provide some useful information to test the validity of the model. For instance, we should obtain an increasing function taking a value of zero at zero.<sup>11</sup> The comparison of the estimated  $\lambda(\cdot)$  functions for different values  $I$  would allow us to assess whether the utility function is independent of the number of bidders. As mentioned previously, such an independence is a natural restriction. Since we have adopted a fully nonparametric approach, such an information will be revealed by the data.

A third step may consist in using the estimated function  $\lambda(\cdot)$  to assess its adjustment with a parametric function of the form  $\lambda(\cdot; \theta)$  derived from a CRRA or CARA model. If we consider a CRRA model,  $U(x) = x^{1-c}$  with  $c \geq 0$  measuring the constant relative risk aversion giving  $\lambda(x; c) = x/(1 - c)$ . In a CARA framework,  $U(x) = [1 - \exp(-ax)]/[1 - \exp(-a)]$ , with  $a > 0$  measuring the constant absolute risk aversion giving  $\lambda(x; a) = (1/a)[\exp(ax) - 1]$ . The parameter  $\theta$  cannot be estimated by a least square estimator because of the correlation between  $\hat{v}_{i\ell} - b_{i\ell}$  and the error term of the following model

$$\frac{1}{I_\ell - 1} \frac{\hat{G}(b_{i\ell}|Z_\ell, I_\ell)}{\hat{g}(b_{i\ell}|Z_\ell, I_\ell)} = \hat{\lambda}(\hat{v}_{i\ell} - b_{i\ell}; \theta) + \epsilon_{i\ell},$$

$i = 1, \dots, I_\ell, \ell = 1, \dots, L_2$ . In particular,  $\hat{v}_{i\ell}$  depends on  $\hat{G}(\cdot|\cdot, \cdot)$ . We propose instead an estimator in the spirit of a semiparametric Generalized Method of Moments (GMM)

---

<sup>11</sup>Note that the function  $\lambda(\cdot)$  can take different shapes. For instance, for a CRRA specification,  $\lambda(\cdot)$  is strictly concave if  $0 \leq c < 1$  and strictly convex if  $c > 1$ .

estimator.<sup>12</sup> In the general case, for any  $b \in (\underline{b}, \bar{b}]$  since  $G(b|z, I)/[(I-1)g(b|z, I)] = \lambda(v-b; \theta)$ , we have  $\theta = \Psi(G(b|z, I)/((I-1)g(b|z, I)), v-b)$ .<sup>13</sup> We then propose to estimate  $\theta$  as follows

$$\hat{\theta} = \frac{1}{N} \sum_{i=1}^{I_\ell} \sum_{\ell=1}^{L_2} \Psi \left( \frac{1}{I_\ell - 1} \frac{\hat{G}(b_{i\ell}|Z_\ell, I_\ell)}{\hat{g}(b_{i\ell}|Z_\ell, I_\ell)}, \hat{v}_{i\ell} - b_{i\ell} \right),$$

where  $N$  is the total number of bids when pooling the data. Let us consider the CRRA case. A CRRA specification is of particular interest as it encompasses the case of risk neutrality, i.e.  $c = 0$  or  $\theta = 1$ . Thus

$$\hat{\theta} = \frac{1}{N} \sum_{i\ell} (I_\ell - 1)(\hat{v}_{i\ell} - b_{i\ell}) \frac{\hat{g}(b_{i\ell}|Z_\ell, I_\ell)}{\hat{G}(b_{i\ell}|Z_\ell, I_\ell)}. \quad (8)$$

We study the asymptotic properties of such an estimator. Let  $[(I_\ell - 1)(\hat{v}_{i\ell} - b_{i\ell})/\hat{G}(b_{i\ell}|Z_\ell, I_\ell)] = w_{i\ell}$ , which is a random variable. Thus  $\hat{\theta} = (1/N) \sum_i \sum_\ell w_{i\ell} \hat{g}(b_{i\ell}|Z_\ell, I_\ell)$ . Since  $\hat{v}_{i\ell}$  and  $\hat{G}(\cdot, \cdot, \cdot)$  converge faster than  $\hat{g}(\cdot|\cdot, \cdot)$ , the consistency rate of  $\hat{\theta}$  is given by the one of  $\hat{g}(\cdot|\cdot, \cdot)$ . To avoid the double sum, we index the observations by  $n$  giving  $\hat{\theta} = (1/N) \sum_{n=1}^N w_n \hat{g}(b_n|Z_n, I_n)$ . The following proposition shows the asymptotic normality of our estimator. Moreover, our semiparametric estimator is  $\sqrt{N}$  consistent.

**Proposition 2:** *Under Assumption A1 (see appendix), we have*

$$\sqrt{N}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, \text{Var}_0\{[w_n + E(w_n|b_n, Z_n, I_n)]g_0(b_n|Z_n, I_n)\}),$$

where  $\theta_0 = E_0[w_n g_0(b_n)]$ .

The assumptions in A1 are quite standard. In particular, they require that  $R+1 > d+1$  and  $Nh_g^{*2(d+1)} \rightarrow \infty$  and  $Nh_g^{*2(R+1)} \rightarrow 0$  as  $N \rightarrow \infty$ . Consequently, the bandwidth  $h_g^*$  used in  $\hat{g}(\cdot|\cdot)$  in (8) should be smaller than the optimal bandwidth resulting in some undersmoothing and in a consistency rate slower than  $N^{(R+1)/(2R+d+3)}$ . The asymptotic

---

<sup>12</sup>See Powell (1994) for a survey on semiparametric estimation.

<sup>13</sup>The function  $\lambda(\cdot; \theta)$  needs to be monotonic in  $\theta$  to allow for a unique solution, which is the case for both CRRA and CARA specifications.

normality of  $\hat{\theta}$  and the derivation of its variance allow us to test for risk neutrality, i.e.  $H_0 : \hat{\theta} = 1$ . The proof of Proposition 2 is given in the appendix. Proposition 2 can extend to the case of a general specification of the utility function.

### 3 Timber Data

As is well known, the US Forest Service (USFS) is selling standing timber from publicly owned forests through both first-price sealed-bid auctions and ascending auctions. Though both auction designs can be found, the USFS is using more frequently ascending auctions despite recommendations by the Congress in 1976 to adopt first-price sealed-bid auctions. With the exception of Hansen (1985) and Athey, Levin and Seira (2004), the abundant previous empirical literature on USFS timber auctions has used data from a single auction design exclusively while addressing important economic issues. Adopting a private value framework, Baldwin, Marshall and Richard (1997) study collusion in ascending auctions while Haile (2001) analyzes the bidding behavior when bidders consider potential resale opportunities after the auction. Athey and Levin (2001) study the practice of skewed bidding when bidders bid on species and when payments are based on actual harvested values. The data analysis conducted in Athey and Levin (2001) suggests that bidders are risk averse as their bidding behavior seems consistent with a diversification of risk among species. Potential bidders' risk aversion has been also found by Baldwin (1995) using a reduced form approach. Because our objective is to focus on the empirical assessment of potential bidders' risk aversion, issues such as collusion and resale markets among others are left aside. Campo, Guerre, Perrigne and Vuong (2006) show the complexity of estimating risk aversion when only first-price sealed-bid auction data are available. Under a parametric specification of the bidders' utility function and a conditional quantile restriction that are both needed to identify bidders' risk aversion, their empirical results show significant risk aversion in USFS auctions.

We use data from the states covering the western half of the US, which includes regions

1 to 6 as labelled by the USFS. A large part of forestry in these regions is publicly owned and represents an important supply of timber in the country. We focus on the auctions organized in 1979, during which 1,796 ascending auctions and 598 first-price sealed-bid auctions were held.<sup>14</sup> Our study focuses on the 1,411 ascending auctions and the 378 sealed-bid auctions in which at least two bidders participated.<sup>15</sup> Note that this large number of auctions is especially attractive when considering nonparametric estimators. Since we consider a general specification for the utility function, we are interested in the total bids for every tract, i.e. the average bid per unit of volume measured in thousand board feet or mbf multiplied by the estimated volume across all the species composing the tract. The USFS provides detailed information on the estimated volume of timber, the number of acres of the parcel, the estimated appraisal value of the timber, the region where the auction took place, the season during which the auction was held, the exact location of the timber parcel, the term of the contract, the logging costs as well as other costs such as road construction costs, the number of bidders who have submitted a bid as well as their bids in dollars and their identity. An unusual characteristic of these auctions is that the number of bidders is recorded for ascending auctions since firms need to submit a qualifying bid prior to the auction. The appraisal value of timber is computed by the USFS taking into account the heterogeneity of timber quality within each parcel. This information is especially useful to the analyst as it provides a good measure of heterogeneity across auctioned objects.

The USFS announces a reserve price at the beginning of the auction. It is well accepted among economists that this reserve price does not act as a screening device as it is set too low. Using the first-price sealed-bid auction data, we have computed the conditional probability that a bid is in the neighborhood of the reserve price, results suggest that the reserve price does not truncate the bid distribution.<sup>16</sup> We can then consider that the

---

<sup>14</sup>For the purpose of comparison with the results obtained by Campo, Guerre, Perrigne and Vuong (2006) using a semiparametric approach, we use the same year and regions as in that paper.

<sup>15</sup>A total of 175 auctions did not receive any bid, while 430 auctions received a single bid.

<sup>16</sup>Specifically, we have estimated nonparametrically the probability  $\Pr(p_0 \leq b \leq (1 + \delta)p_0 | Z)$ , where

reserve price is nonbinding as a reasonable approximation.<sup>17</sup> Table 1 gives some basic statistics on some key variables. The bids, winning bids and appraisal values are given in dollars, while the volume is given in mbf.<sup>18</sup>

**Table 1:** Summary Statistics

	Ascending Auctions		Sealed-Bid Auctions	
	$L = 1,411$		$L = 378$	
	Mean	STD	Mean	STD
Winning Bids	1,309,876.48	1,939,234.25	211,638.75	520,461.78
Bids	–	–	202,563.66	494,178.42
Volume	4,998.23	5,507.21	1,624.76	3,152.53
Acres	1,129.08	4,749.46	1,348.64	3,590.59
Appraisal Value	419,845.50	556,259.29	76,864.92	226,561.62
Number of Bidders	5.35	2.87	3.72	1.81

We note some important differences in the tract characteristics between the ascending and sealed-bid auctions. The volume of timber is on average three times larger in ascending auctions than in sealed-bid auctions though displaying less variability in ascending auctions than in sealed-bid auctions as measured by the coefficient of variation.

---

$p_0$  is the reserve price,  $Z$  is the total appraisal value and  $\delta$  an arbitrary value larger than 0. For an average appraisal value, we find that this probability is equal to 1.4% for  $\delta = 0.05$  and 4.5% for  $\delta = 0.10$  suggesting that the truncation on the bid distribution due to the reserve price is minor. See also Haile (2001) who provides several arguments in favor of nonbinding reserve prices.

<sup>17</sup>The relatively large number of auctions which did not receive any bid might suggest the opposite. A further analysis of the number of participants shows that some exogenous characteristics explain bidders' participation.

<sup>18</sup>The highest bid is recorded by the USFS for every bidder in ascending auctions. Since the format used in ascending auctions is quite different from the theoretical model of ascending auctions, we consider only the winning bid in the ascending auctions because only the winning bid can be considered as relevant and can be approximated by the theoretical auction model. Such an assumption has been made in other empirical studies involving ascending auction data.

Consequently, the mean for the total appraisal value is quite different between the two auction formats. This difference is not only due to a difference in volume of timber. When computing the mean for the appraisal value per unit of volume, we find it equal to 89.34 in ascending auctions and to 57.23 in sealed-bid auctions suggesting that the quality of timber is higher in tracts sold through ascending than sealed-bid auctions. This difference can also be explained by a larger density of timber per acre in parcels sold through ascending auctions than in parcels sold through first-price sealed-bid auctions, 15.96 and 7.26, respectively. As a matter of fact, parcels are almost of the same size on average between the two auctions. To summarize, the parcels sold through ascending auctions involve on average a higher volume of timber, a better quality of timber and a higher density of timber than parcels sold through first-price sealed-bid auctions for a similar area. As expected, the winning bids reflect these differences.<sup>19</sup> Ascending auctions tend to attract on average more bidders than sealed-bid auctions, 5.35 and 3.72, respectively. This suggests that the number of bidders is a function of the lot characteristics. It is interesting to investigate further these two issues, namely the choice of the auction format and the number of participants since both depend on lot characteristics.<sup>20</sup>

A simple probit model assessing the impact of some exogenous variables on the probability to choose an ascending auction over a sealed-bid auction confirms that the volume plays an important role with a highly significant coefficient, while the coefficient for the density of timber is also significant to a much less extent. The coefficient for the appraisal value measured by unit of volume of timber is insignificant. The results also show that some regions are more likely to adopt ascending auctions than sealed-bid auctions such as regions 2, 3 and 6.<sup>21</sup> Other variables such as the quarter dummies during which auctions were held do not provide significant coefficients. The pseudo  $R^2$  is equal to 0.87, showing

---

<sup>19</sup>Table 1 reports only the main variables. Table 2 will further show that appraisal value, volume and density explain more than 90% of the variability in the winning bids through a linear regression.

<sup>20</sup>Detailed results are available upon request to the authors.

<sup>21</sup>We do not observe any sealed-bid auction in Region 2.

a respectable adjustment.

We also estimate a regression model to assess the factors explaining the number of participants including the auctions with no bidder and one bidder. This gives a total of 1,796 ascending auctions and 598 sealed-bid auctions. We consider a wide range of variables such as the appraisal value per mbf, the total volume, the timber density, region and season dummies as well as a dummy for ascending auction when data from both auction formats are combined. We do not include the reserve price in this regression model since it is highly correlated with the appraisal value with a coefficient of correlation equal to 0.92. For sealed-bid auctions, larger volume of timber, auctions organized in Region 6 and auctions during the spring season tend to attract more bidders. The pattern is somewhat different for ascending auctions. Larger volume of timber, larger density of timber, auctions organized in Region 5 and auctions organized during the winter season tend to attract a larger number of bidders. Note that the  $p$ -values for volume and timber density are less than 0.005 for ascending auctions giving strongly significant coefficients. The  $R^2$  of such a regression is about 0.40 for ascending auctions and 0.20 for sealed-bid auctions suggesting some unobserved heterogeneity. An interesting feature of this regression when pooling all the data is that the dummy for the auction format is insignificant while controlling for other exogenous factors. This suggests that there is a priori no self-selection of bidders in the sense that the auction format does not affect the decision of a bidder to participate. This issue is crucial to our problem since we use the ascending auction bidding data to estimate the latent density of bidders' private value. Because of the importance of this issue, we have also used information on bidders' identity given by the data. In every region except Region 2 (see footnote 21), we have tracked independently over the two auction samples twenty companies according to their high participation frequency. We have then compared for every region these two lists, each containing the identity of twenty firms. We find an important intersection in the sense that very few of these firms participate exclusively to ascending or sealed-bid auctions. Consequently, we can consider as a reasonable approximation that, despite ascending auctions tend to attract more bid-

ders than sealed-bid auctions, the set of bidders is similar across the two auction formats and that bidders' decision participation seems to be independent of the auction format but rather influenced by the tract (observed and unobserved) characteristics.<sup>22</sup>

**Table 2:** Winning Bids

	Ascending Auctions		Sealed-Bid Auctions	
	Coeff.	<i>t</i> -value	Coeff.	<i>t</i> -value
Constant	1.5862	18.06	2.0132	12.97
Appraisal Value	0.5283	39.24	0.4291	19.57
Volume	1.0824	119.50	1.0664	65.97
Density	0.0056	0.53	0.0087	0.63
# Bidders	0.4651	16.27	0.4712	8.67
$R^2$	0.9406		0.9311	

While considering auctions with more than 2 bidders, Table 2 provides the results of the regression of the logarithm of the total winning bids on the logarithm of the appraisal value per mbf, the logarithm of the total volume, the logarithm of the density and the logarithm of the number of bidders.<sup>23</sup> The estimated coefficients can be interpreted as elasticities. They do not vary much across the two auction formats except for the appraisal value, namely an 1% increase in the appraisal value increases the winning bid by 0.53% in ascending auctions and by 0.43% only in sealed-bid auctions everything else being equal. The elasticity with respect to density is insignificant in both auctions. Winning bids are quite sensitive to a variation in competition.<sup>24</sup> A Chow test weakly rejects the equality of the slope coefficients. The fact that the winning bids are roughly explained

<sup>22</sup>We focus on the twenty most important players in these auctions for every region based on their participation frequency. Some of these firms also participate to auctions in other regions in proximity. On the other hand, the data show a large number of firms participating to a single auction.

<sup>23</sup>We do not include in this regression season and region dummies as they do not provide significant coefficients.

<sup>24</sup>This result is confirmed when computing for sealed-bid auctions the ratio of the difference between the winning bid and the second highest bid by the winning bid. This ratio is on average equal to 0.16 and



by the same variables with similar coefficient magnitudes across the two auction designs suggests that the set of participants is not different. On the other hand, the constant differs significantly across the two regressions. As discussed previously, the number of participants could capture some unobserved heterogeneity across tracts. The relatively large  $R^2$  in Table 2 suggests that the heterogeneity across auctioned tracts is quite well captured with the appraisal value, the volume and the number of bidders. The estimation in the following section is performed while conditioning on the appraisal value, the volume and the number of participants.

**Table 3:** Sample Selection on Winning Sealed-Bids

	Sealed-Bid Auctions	
	Coeff.	$t$ -value
Constant	2.2570	11.34
Appraisal Value	0.3819	12.18
Volume	1.0167	30.69
Density	-0.0232	-0.98
# Bidders	0.4394	7.56
Sample Selection	0.6157	1.75

The summary statistics in Table 1 suggest some sample selection as parcels with a larger volume of timber are sold through ascending auctions. We perform a sample selection correction following Heckman selection model in two steps. In a first step, we estimate a probit with the dependent variable equal to one for sealed-bid auctions. We find that parcels with larger volume, density and appraisal value are less likely to be sold through sealed-bid auctions. The regression of the logarithm of the winning bids for sealed-bid auctions with sample selection are given in Table 3. The coefficient for sample

---

sharply decreases when the number of bidders increases. As a matter of fact, auction theory predicts that bids increase in the number of bidders within the independent private value paradigm. Such a result still holds with risk aversion. With affiliated private values, bids may decrease with the number of bidders. See Pinkse and Tan (2005). Thus our results would favor the independence of private values.

selection is significant at 10% only with a  $p$ -value equal to 0.08. Special attention should be given to the conditional variables in the estimation.

It is interesting to make the parallel with the results obtained by Athey, Levin and Seira (2004) who use ascending and sealed-bid sales in two forests in Northern region and in California over 1982-1990. Their empirical analysis of the data shows that the ascending auction format is more likely to be chosen for larger volumes as in our data set. However, the number of participants across the two auctions does not vary as much as in our data set. They collected additional data on the participating firms, on whether they are loggers or mills, mills having manufacturing capacity and consequently considered as large bidders. When taking into account this asymmetry across firms, mills are more likely to participate to ascending auctions while loggers are more likely to participate to sealed-bid auctions. Their paper exploits this bidders' heterogeneity to model endogenous bidders' participation and to rationalize the fact that sealed bidding favors small bidders or loggers in both entry and allocation, while generating more revenue. We did not collect such data on bidders though our identification result could allow for bidders' heterogeneity. We find in our data set, which encompasses more regions over a single year, that in every region the most active firms are participating to both auctions.

## 4 Estimation Results

The first step of our estimator consists in estimating the conditional distribution of private values  $F(\cdot|\cdot, I)$ . We need first to discuss the choice of the vector of characteristics  $Z$ . The results in Table 2 clearly show that the volume and to a lesser extent the appraisal value explain an important proportion of the variability of the winning bids in ascending auctions. Thus we consider a two-dimension vector of exogenous variables  $Z$  with total volume and appraisal value per unit of volume. We perform the nonparametric estimation of  $F(\cdot|\cdot, \cdot)$  for a given size of bidders  $I$  as we do not restrict ourselves to exogenous bidders' participation. The data provide 241 auctions with 2 bidders, 231 auctions with 3 bidders,

189 auctions with 4 bidders, 178 auctions with 5 bidders, 141 auctions with 6 bidders and so on. The sealed-bid auction data do not provide, however, such large numbers of auctions, namely 107 auctions with 2 bidders, 108 auctions with 3 bidders, 58 and 54 auctions with 3 and 4 bidders, respectively. Given the small number of sealed-bid auctions with 3 and 4 bidders, we consider auctions with 2 and 3 bidders.

Because of the use of nonparametric methods, the estimation results for conditional distributions are mainly presented through graphs. The two-dimension vector of characteristics makes difficult any graphical representation of distributions and densities. Thus we need to choose some values for  $Z$  to represent the conditional bid and private value distributions. Moreover, the range of values for volume is quite different for ascending and sealed-bid auctions with a mean almost three times larger for ascending auctions than for sealed-bid auctions, while the appraisal value is on average about 50% more for ascending than for sealed-bid auctions. Thus we need to check whether we have enough observations for ascending auctions in the range of values for sealed-bid auctions. For this purpose, we present the scatter plots of volume versus appraisal value for both auction data for  $I = 2$  and  $I = 3$  in Figures 1 and 2, respectively. Note that Figures 1 and 2 do not contain all the observations to facilitate the interpretation of such figures. We observe an important concentration of observations for volumes between 100 and 1,000 and for appraisal values between 10 and 150. When considering the full sample, the median for appraisal value for sealed-bid auctions is equal to 43.65, while the median value for volume is equal to 490 indicating an important skewness for the latter (see also Table 1). Figures 1 and 2 show that despite displaying on average larger volume and appraisal values, ascending auction data provide a significant number of observations in the range of values discussed previously. Another difficulty arises from the large dispersion and the large range of values for the volume in ascending auctions relative to sealed-bid auctions. Thus, we need to reduce the range of values for volume on which we estimate the conditional private value distribution. We propose to consider the 90% percentile of the volume for sealed-bid auctions as the cut-off point for ascending auctions. These quantiles are equal to 1,860

and 3,086 for  $I = 2$  and  $I = 3$ , respectively. This leads us to exclude 99 and 74 ascending auctions from the estimation of the conditional private value distribution for  $I = 2$  and  $I = 3$ , respectively. By doing so, we obtain estimates for the conditional distributions while solving for the problem of different ranges of values for the conditioning variables.

We estimate nonparametrically the distributions  $F(\cdot|z, I = 2)$  and  $F(\cdot|z, I = 3)$  with  $z = (z_1, z_2) = (490.00, 43.65)$  corresponding to the median values for volume and appraisal values for sealed-bid auctions. These estimates are displayed on Figure 3. Using the distributions of order statistics, we can also estimate the density. In particular, for  $I = 2$   $f^{1,2}(v|z, 2) = 2f(v|z, 2)(1 - F(v|z, 2))$ . We can estimate nonparametrically  $f^{1,2}(v|z, N)$  using a kernel estimator as in (7). Thus  $\hat{f}(\cdot|z, 2) = \hat{f}^{1,2}(\cdot|z, 2)/[2(1 - \hat{F}(\cdot|z, 2))]$ , where  $\hat{F}(\cdot|z, 2)$  denotes the estimated distribution obtained in the first step of our estimation procedure,  $\hat{f}^{1,2}(\cdot|z, 2)$  denotes the nonparametric estimate of  $f^{1,2}(\cdot|z, I)$ . Similarly for  $I = 3$ . Figure 4 displays both conditional densities with the conditional density for  $I = 2$  more peaked than for  $I = 3$ . We have performed the estimation at other conditioning values for volume and appraisal values and still obtained different conditional densities of private values. Thus these results suggest that the distribution of private values differ across the number of bidders. As discussed previously, the number of bidders may capture some unobserved heterogeneity.

The next step consists in estimating nonparametrically the bid distribution  $G(\cdot|z, I)$  using (6). Figures 5 and 6 display the conditional distributions of private values and bids for  $I = 2$  and  $I = 3$ , respectively. An interesting feature of these figures is that they allow us to test a first prediction of the model. In particular, since  $v > b$  on  $(0, s(\bar{v})]$ , the bid distribution should stochastically dominate the private value distribution. We observe such a stochastic dominance in both figures. The Kolmogorov test statistic leads to reject the equality of distributions for  $I = 2$  and  $I = 3$ .<sup>25</sup> As discussed previously, this test represents a first step toward the validity of the model. From the estimated distribution  $\hat{G}(\cdot|\cdot, \cdot)$ , we can estimate for every observation  $b_{i\ell}$  (for auctions with two and three

---

<sup>25</sup>A similar exercise has been conducted at different values of  $z$  leading to the same conclusion.

bidders) the corresponding  $\alpha$ -quantile such that  $\hat{b}(\alpha, Z_\ell, I_\ell) = b_{i\ell}$ . From the estimated distribution  $\hat{F}(\cdot|\cdot, \cdot)$ , we can then estimate the corresponding  $\alpha$ -quantile  $\hat{v}(\alpha, Z_\ell, I_\ell)$ . This value provides the estimated private value  $\hat{v}_{i\ell}$ . It remains to estimate the bid density using (7). This gives  $\hat{g}(b_{i\ell}|Z_\ell, I_\ell)$ . This information will allow us to recover  $\lambda(\cdot)$ . To avoid boundary effects due to the use of kernel estimators, we propose a simple trimming rule as follows. We estimate  $\lambda(\cdot)$  while using only observations  $b_{i\ell}$  corresponding to the conditional  $\alpha$ -quantiles with  $\alpha \in [0.05, 0.95]$ . Figures 7 and 8 display the scatter plot of the pairs  $(\hat{v}_{i\ell} - b_{i\ell}, \hat{G}(b_{i\ell}|Z_\ell, I_\ell)/[(I-1)\hat{g}(b_{i\ell}|Z_\ell, I_\ell)])$  for  $I = 2$  and  $I = 3$ , respectively. The continuous line represents the smoothed function. These figures display as well the  $\lambda(\cdot)$  function obtained for the parameters for CRRA and CARA specifications of the utility function using the estimator defined in (8). The (bold) dashed line represents the (CRRA) CARA estimated model. These figures do not allow us to check whether  $\lambda(0) = 0$  as we cannot estimate properly this function close to the boundaries. We remark, however, that the displayed scatter plots form a shape, which could reasonably go to zero at zero.

Using the estimated  $\lambda(\cdot)$ , we can recover the function  $U(\cdot)$ , whose shape is more informative since its concavity would indicate bidders' risk aversion. Figures 9 and 10 display such utility functions as well as the estimated CRRA and CARA utility functions for  $I = 2$  and  $I = 3$ , respectively. Because the scale is irrelevant, we have normalized the utility function to be in the interval  $[0, 1]$ . The displayed utility functions are increasing and concave indicating risk aversion. We obtain  $\hat{c} = 0.5928$  for a CRRA specification and  $\hat{a} = 0.00004$  for a CARA specification when  $I = 2$ . Figures 7 and 9 suggest that the CRRA specification provides a better fit with a tendency to overestimate the utility function. For  $I = 3$ , we obtain  $\hat{c} = 0.5994$  for a CRRA specification and  $\hat{a} = 0.000042$  for a CARA specification. We note that the values for risk aversion parameters do not vary much from the case  $I = 2$ . As for  $I = 2$ , a CRRA specification provides a better fit while we observe the same tendency, i.e the CRRA specification tends to overestimate the utility function. In particular, the estimated CRRA utility function is somewhat above the nonparametrically estimated utility function, while the CARA utility function tends

to give larger values of utility. This discrepancy tends, however, to reduce for larger rent values. When comparing the estimated utility functions for  $I = 2$  and  $I = 3$ , we observe that they are almost identical. It is interesting to test for risk neutrality using the estimation results for a CRRA specification. In particular,  $c = 0$  or  $\theta = 1$  corresponds to risk neutrality. Using Proposition 2, we can perform such a test. The  $t$ -values for testing  $H_0 : \hat{\theta} = 1$ , which are equal to -11.86 for  $I = 2$  and -7.71 for  $I = 3$  clearly reject risk neutrality in both cases.

We have performed the estimation of the private value distribution using the results of Campo, Guerre, Perrigne and Vuong (2006) under a CRRA specification and a linear upper bound for the private value distribution. In particular, the private values are recovered from  $\hat{v}_{i\ell} = b_{i\ell} + [(\theta \hat{G}(b_{i\ell}|Z_\ell, I_\ell))/((I-1)\hat{g}(b_{i\ell}|Z_\ell, I_\ell))]$ , with  $\theta = 0.6813$ . We have then estimated the conditional distribution of private values for  $z = (z_1, z_2) = (490.00, 43.65)$ . We have performed the same exercise with  $\theta = 1$  (risk neutrality). Figures 11 and 12 display such conditional private value distributions as well as  $\hat{F}(\cdot|z, I)$  obtained from ascending auctions for  $I = 2$  and  $I = 3$ , respectively.<sup>26</sup> The difference between our estimated private value distribution with the one obtained with  $\theta = 1$  can be explained by the presence of risk aversion as tested above. The difference with the private value distribution from Campo, Guerre, Perrigne and Vuong (2006) can be explained by various factors. Campo, Guerre, Perrigne and Vuong (2006) tend to underestimate risk aversion relative to our results. This may be due to a misspecification issue on the upper bound of  $F(\cdot)$  in this paper. Their results are obtained while (i) conditioning on the total appraisal value (instead of the volume and the appraisal value per mbf in our case), (ii) pooling the data across the different number of bidders, and (iii) excluding 79 auctions because of some outliers causing problems in the estimation of the upper boundary of the bid distribution. It is,

---

<sup>26</sup>In Figures 11 and 12, the upper distribution has been estimated from ascending auction data. The middle distribution has been estimated from sealed-bid bid auction data and the CRRA risk aversion estimate in Campo, Guerre, Perrigne and Vuong (2006). The lower distribution has been estimated from sealed-bid auction data, while imposing risk neutrality.

however, possible that, despite our efforts to check for this issue, the ascending auction data do not provide an exact representation of the bidders' private value distribution in sealed-bid auctions.

## 5 Conclusion

The paper shows how to identify nonparametrically the bidders' utility function when bidding data from both ascending and first-price sealed-bid auction formats are available to the analyst. The utility function, which is at the core of decisions under uncertainty has not been estimated nonparametrically so far on data. As a matter of fact, little is known in practice on how agents evaluate their gain when facing uncertainties. Relying on bidding data from the timber sales at the US Forest Service, our empirical results show that the recovered utility function is increasing while displaying some concavity. A CRRA specification partly captures bidders' risk aversion.

Our identifying strategy exploits data from two auction formats, which can be quite restrictive as institutions use in general a single auction design. Guerre, Perrigne and Vuong (2006) exploit some exclusion restrictions such as the independence of the private value distribution upon the number of bidders to identify nonparametrically the utility function. Their identification result requires the construction of a series of differences in quantiles, which are serially correlated. This feature greatly complicates the asymptotic properties of such an estimator though its implementation on data could be performed. It would then be interesting to compare our results to those obtained using such a result. Such empirical comparisons would allow us to assess the relevance of various restrictions used to identify the bidders' utility function.

## Appendix

The appendix provides Assumptions A1 under which Proposition 2 holds as well as the proof of Proposition 2. The assumptions and the proof follow the assumptions and proofs of Vuong (2003) regarding the average density and Powell, Stock and Stocker (1989, Theorem 3.2) regarding average derivatives. To simplify the proof, we omit the conditioning on  $(Z, I)$ . We can easily extend Assumption A1 and the proof of Proposition 2 to include a vector  $(Z, I)$  of exogenous variables.

**Assumptions A1:** Let  $(W, B) \in \mathbb{R}^2$  distributed with a joint density  $\phi(\cdot, \cdot)$ . Suppose that (i) For  $R + 1 \geq 2$ , the density  $\phi_0(\cdot, \cdot)$  is  $R + 1$  continuously differentiable on  $\mathbb{R}^2$ . The conditional density  $\phi_0(B|W)$  has uniformly bounded derivatives, i.e.  $\|\phi_0^{(s)}\| < M < \infty$  for  $0 \leq s \leq R + 1$ . Moreover,  $E_0[W^2]$  and  $E[W]$  are finite.

(ii) The function  $K(\cdot)$  is an  $(R+1)$ th order kernel on  $\mathbb{R}$ , i.e.  $\int K(u)du = 1$ ,  $\int K(u)u^s du = 0$  for  $1 \leq s < R + 1$  and  $\int |K(u)||u|^{R+1}du$  is finite. Moreover,  $\|K(\cdot)\|$  and  $\int K^2(u)du$  are finite.

(iii)  $Nh^4 \rightarrow \infty$  and  $Nh^{2(R+1)} \rightarrow 0$  when  $N \rightarrow \infty$ .

**Proof:** The proof is based on U-statistics. We have

$$\hat{\theta} = \frac{1}{N^2} \sum_{n=1}^N W_n K_h(0) + \frac{1}{N^2} \sum_{n=1}^N \sum_{\tilde{n}=1, \tilde{n} \neq n}^N N W_n K_h(B_n - B_{\tilde{n}}) \equiv R_N + \frac{N(N-1)}{N_2} U_N,$$

where  $K_h(u) = (1/h)K(u/h)$ . Because  $K_h(0) = (1/h)K(0)$ , it gives

$$R_N = \frac{1}{N_2} \sum_{n=1}^N W_n \frac{1}{h} K(0) = O_p\left(\frac{1}{Nh}\right) = o\left(\frac{1}{\sqrt{N}}\right),$$

because  $\sqrt{N}h \rightarrow \infty$  and by A1-(iii). The term  $U_N$  can be written as a U-statistic, namely

$$\begin{aligned} U_N &= \frac{1}{N(N-1)} \sum_{n=1}^N \sum_{\tilde{n}=1, \tilde{n} \neq n}^N W_n K_h(B_n - B_{\tilde{n}}) \\ &= \frac{1}{N(N-1)} \sum_{n=1}^{N-1} \sum_{\tilde{n}=n+1}^N [W_n K_h(B_n - B_{\tilde{n}}) + W_{\tilde{n}} K_h(B_{\tilde{n}} - B_n)] \end{aligned}$$



$$= \binom{N}{2}^{-1} \sum_{n=1}^{N-1} \sum_{\tilde{n}=n+1}^N p_N(Y_n, Y_{\tilde{n}}),$$

where  $Y_n = (W_n, B_n)$  and  $Y_{\tilde{n}} = (W_{\tilde{n}}, B_{\tilde{n}})$ . Let  $\hat{U}_N$  be the projection of  $U_N$  of  $\{Y_1, \dots, Y_N\}$  (see Serfling (1980))

$$\hat{U}_N \equiv \theta_N + \frac{2}{N} \sum_{n=1}^N (r_N(Y_n) - \theta_N),$$

where  $r_N(Y_n) = E_0[p_N(Y_n, Y_{\tilde{n}})|Y_n]$  and  $\theta_N = E_0[r_N(Y_n)] = E_0[p_N(Y_n, Y_{\tilde{n}})]$  for  $\tilde{n} \neq n$ .

STEP 1: We need to show that  $\sqrt{N}(U_N - \hat{U}_N) = o_p(1)$ . By Lemma 3.1 in Powell, Stock and Stoker (1989), it suffices to show that  $E_0[p_N^2(Y_n, Y_{\tilde{n}})] = o(N)$ . Using the change of variable  $u = (b_{\tilde{n}} - b_n)/h$ , it gives

$$\begin{aligned} E_0[p_N^2(Y_n, Y_{\tilde{n}})] &= \frac{1}{4} \iiint [w_n K_h(b_n - b_{\tilde{n}}) + w_{\tilde{n}} K_h(b_{\tilde{n}} - b_n)]^2 \phi_0(b_n, w_n) \phi_0(b_{\tilde{n}}, w_{\tilde{n}}) db_n dw_n db_{\tilde{n}} dw_{\tilde{n}} \\ &= \frac{1}{4} \iiint \frac{1}{h^4} \left[ w_n K\left(\frac{b_n - b_{\tilde{n}}}{h}\right) + w_{\tilde{n}} K\left(\frac{b_{\tilde{n}} - b_n}{h}\right) \right]^2 \phi_0(b_n, w_n) \phi_0(b_{\tilde{n}}, w_{\tilde{n}}) db_n dw_n db_{\tilde{n}} dw_{\tilde{n}} \\ &= \frac{1}{4h^2} \iiint [w_n K(-u) + w_{\tilde{n}} K(u)]^2 \phi_0(b_n, w_n) \phi_0(b_n + uh, w_{\tilde{n}}) db_n dw_n du dw_{\tilde{n}} \\ &\leq \frac{1}{2h^2} \iiint [w_n^2 K^2(-u) + w_{\tilde{n}}^2 K^2(u)] \phi_0(b_n, w_n) \phi_0(b_n + uh, w_{\tilde{n}}) db_n dw_n du dw_{\tilde{n}}, \end{aligned}$$

because  $(a + b)^2 \leq 2(a^2 + b^2)$ . Since  $\phi_0(b_n + uh|w_{\tilde{n}})\phi_0(w_{\tilde{n}}) = \phi_0(b_n + uh, w_{\tilde{n}})$ , the last term can be written as

$$\frac{1}{2h^2} \int \int \int [w_n^2 K^2(-u) + w_{\tilde{n}}^2 K^2(u)] \phi_0(b_n, w_n) \phi_0(b_n + uh|w_{\tilde{n}}) \phi_0(w_{\tilde{n}}) db_n dw_n du dw_{\tilde{n}}.$$

Since  $\phi(b_n + uh|w_{\tilde{n}})$  is bounded from infinity by (say)  $M$  and  $\int K^2(-u)du = \int K^2(u)du$ , we need to show that  $K(u)[w_n^2 + w_{\tilde{n}}^2]\phi_0(b_n, w_n)\phi_0(w_{\tilde{n}})$  is integrable with respect to  $b_n, u, w_n$  and  $w_{\tilde{n}}$ , i.e. the integral takes a finite value. We can easily perform the integration with respect to  $b_n$ , the term  $\phi_0(b_n, w_n)$  becomes  $\phi_0(w_n)$ . Moreover,  $\|K(\cdot)\| < \infty$  by A1-(ii). Thus it remains  $\int w_n^2 \phi_0(w_n) dw_n + \int w_{\tilde{n}}^2 \phi_0(w_{\tilde{n}}) dw_{\tilde{n}}$ . Consequently, the dominating function is integrable. Using the Lebesgue Dominated Convergence Theorem, when  $h \rightarrow 0$ , the

quadruple integral converges to  $\int K^2(u)du \int \int [w_n^2 + w_{\bar{n}}^2] \phi_0(b_n, w_n) \phi_0(b_n, w_{\bar{n}}) db_n dw_n dw_{\bar{n}} < \infty$ . Thus  $E_0[p_N^2(Y_n, Y_{\bar{n}})] = (1/2h^2)O(1) = o(N)$  since  $Nh^2 \rightarrow \infty$  by A1-(iii).

STEP 2: We need to show that  $\sqrt{N}(\hat{U}_N - \theta_N) \xrightarrow{d} \mathcal{N}(0, \text{Var}_0\{[w_n + E(w_n|b_n)]g_0(b_n)\})$ .

Using the change of variable  $u = (b_{\bar{n}} - B_n)/h$ , we have

$$\begin{aligned} r_N(Y_n) &= \frac{1}{2h^2} \int \int \left[ W_n K\left(\frac{B_n - b_{\bar{n}}}{h}\right) + w_{\bar{n}} K\left(\frac{b_{\bar{n}} - B_n}{h}\right) \right] \phi_0(b_{\bar{n}}, w_{\bar{n}}) db_{\bar{n}} dw_{\bar{n}} \\ &= \frac{1}{2} \left[ \int \int W_n K(u) \phi_0(B_n - uh, w_{\bar{n}}) dudw_{\bar{n}} + \int \int w_{\bar{n}} K(u) \phi_0(B_n + uh, w_{\bar{n}}) dudw_{\bar{n}} \right] \\ &= \frac{1}{2} [W_n + E_0(W_n|B_n)] \int \phi_0(B_n, w_{\bar{n}}) dw_{\bar{n}} + t_N(Y_n). \end{aligned}$$

Note that  $\int \phi_0(B_n, w_{\bar{n}}) dw_{\bar{n}} = g_0(B_n)$  and  $E_0[(1/2)[W_n + E_0(W_n|B_n)]g_0(B_n)] = \theta_0$ . The term  $t_N(Y_n)$  is defined as

$$\begin{aligned} t_N(Y_n) &= \frac{1}{2} \left( \int \int K(u) [W_n \phi_0(B_n - uh, w_{\bar{n}}) - W_n \phi_0(B_n, w_{\bar{n}})] dudw_{\bar{n}} \right. \\ &\quad \left. + \int \int K(u) [w_{\bar{n}} \phi_0(B_n + uh, w_{\bar{n}}) - E_0(W_n|B_n) \phi_0(B_n, w_{\bar{n}})] dudw_{\bar{n}} \right). \end{aligned}$$

Moreover,  $\theta_N \equiv E_0[r_N(Y_n)] = \theta_0 + E_0[t_N(Y_n)]$ . The idea is to show that the variance of  $t_N(Y_n)$  is equal to zero. By definition of  $\sqrt{N}\hat{U}_N$ ,  $\sqrt{N}(\hat{U}_N - \theta_N) = (2/\sqrt{N}) \sum_{n=1}^N [r_N(Y_n) - \theta_N]$  and  $r_N(Y_n) = (1/2)[W_n + E_0(W_n|B_n)]g_0(B_n) + t_N(Y_n)$ . This gives

$$\sqrt{N}(\hat{U}_N - \theta_N) = \frac{2}{\sqrt{N}} \sum_{n=1}^N \left\{ \frac{1}{2} [W_n + E_0(W_n|B_n)]g_0(B_n) - \theta_0 \right\} + \frac{2}{\sqrt{N}} \sum_{n=1}^N [t_N(Y_n) - E_0[t_N(Y_n)]],$$

where the second term is denoted  $T_N$ . We have  $\text{Var}_0(T_N) = 4\text{Var}_0[t_N(Y_n)]$  because the  $Y_n$ s are i.i.d. Note that  $\text{Var}_0(T_N) = E(t_N^2) - E^2(t_N)$ . Since  $E_0^2(t_N) \geq 0$ , we have  $4\text{Var}_0[t_N(Y_n)] \leq 4E_0[t_N^2(Y_n)]$ . Using  $(a+b)^2 \leq 2(a^2 + b^2)$ , we have

$$\begin{aligned} 4E_0[t_N^2(Y_n)] &\leq 2E_0 \left\{ \int \int K(u) [W_n \phi_0(B_n - uh, w_{\bar{n}}) - W_n \phi_0(B_n, w_{\bar{n}})] dudw_{\bar{n}} \right\}^2 \\ &\quad + 2E_0 \left\{ \int \int K(u) [w_{\bar{n}} \phi_0(B_n + uh, w_{\bar{n}}) - E_0(W_n|B_n) \phi_0(B_n, w_{\bar{n}})] dudw_{\bar{n}} \right\}^2. \end{aligned}$$

We study the first and second term of the dominating term. To do so, we consider the  $(R+1)$  Taylor expansions of  $\phi_0(B_n + uh, w_{\bar{n}})$  and  $\phi_0(B_n - uh, w_{\bar{n}})$  around

$B_N$ . Note that the terms involving the derivatives of  $\phi_0(\cdot, \cdot)$  with respect to its first argument up to the  $R$ th derivative disappear because of the kernel, which is of order  $R + 1$  by A1-(ii). We look at the argument of the first term. It remains the term  $\int \int K(u) W_n \phi_0^{(R+1)}(B_n^*, w_{\tilde{n}}) [(-uh)^{R+1}/(R+1)!] du dw_{\tilde{n}}$  where  $B_n - uh < B_n^* < B_n$ . Using the Bayes rule, taking the absolute value and A1-(ii) imply that the argument of the first term is dominated by  $(M/(R+1)!) h^{R+1} w_n \int |K(u)| |u|^{R+1} du$  since  $\int \phi_0(w_{\tilde{n}}) dw_{\tilde{n}} = 1$ . By A1-(iii),  $\int |K(u)| |u|^{R+1} du$  is finite. Thus the first term is dominated by (say)  $(\tilde{M}/(R+1)!) h^{2(R+1)} E_0[W_n^2]$ . By A1-(i),  $E_0[W_n^2]$  is finite. While taking the square, we have a term, which is  $O(h^{2(R+1)})$ . We look at the argument of the second term. Using similar arguments, it remains  $(h^{R+1}/(R+1)!) \int \int K(u) u^{R+1} w_{\tilde{n}} \phi_0^{(R+1)}(B_n^{**}|w_{\tilde{n}}) \phi_0(w_{\tilde{n}}) du dw_{\tilde{n}}$  where  $B_n < B_n^{**} < B_n + uh$ . While taking the absolute value this term is dominated by  $(h^{R+1}/(R+1)!) \int \left[ \int \phi_0^{(R+1)}(B_n^{**}|w_{\tilde{n}}) |K(u)| |u|^{R+1} du \right] w_{\tilde{n}} \phi_0(w_{\tilde{n}}) dw_{\tilde{n}}$ . Since  $\|\phi_0^{(R+1)}(B|w)\|$ ,  $\int |K(u)| |u|^{R+1} du$  and  $E[w_n]$  are finite by A1 (i)-(ii), we have  $(h^{R+1}/(R+1)!) O(1)$ . While taking the square, it gives  $O(h^{2(R+1)})$ . Thus we have shown that  $\text{Var}_0[T_N] = 4\text{Var}_0[t_n(Y_n)] \leq O(h^{2(R+1)})$ . By Chebyshev's inequality,  $T_N = o_p(1)$  as  $h \rightarrow \infty$ . Therefore,

$$\begin{aligned} \sqrt{N}(\hat{U}_N - \theta_N) &= \frac{2}{\sqrt{N}} \sum_{n=1}^N \left\{ \frac{1}{2} [w_n + E_0(w_n|B_n)] g_0(B_N) - \theta_0 \right\} + o_p(1) \\ &\xrightarrow{d} \mathcal{N}(0, \text{Var}_0\{[w_n + E_0(w_n|B_n)] g_0(B_n)\}). \end{aligned}$$

STEP 3: We need to show that  $\sqrt{N}(\theta_N - \theta_0) = o(1)$ . Using the change of variable  $u = (b_{\tilde{n}} - b_n)/h$ , we have

$$\begin{aligned} \theta_N &= E_0[r_N(Y_n)] \\ &= \frac{1}{2} \int \int \int \int \frac{1}{h^2} \left[ w_n K\left(\frac{b_n - b_{\tilde{n}}}{h}\right) + w_{\tilde{n}} K\left(\frac{b_{\tilde{n}} - b_n}{h}\right) \right] \phi_0(b_{\tilde{n}}, w_{\tilde{n}}) \phi_0(b_n, w_n) db_{\tilde{n}} dw_{\tilde{n}} db_n dw_n \\ &= \frac{1}{2} \int \int \int \int [w_n K(-u) + w_{\tilde{n}} K(u)] \phi_0(b_n, w_n) \phi_0(b_n + uh, w_{\tilde{n}}) du dw_{\tilde{n}} db_n dw_n. \end{aligned}$$

While using a Taylor expansion for  $\phi_0(b_n + uh, w_{\tilde{n}})$  around  $b_n$ , the terms involving the derivatives of  $\phi_0(\cdot, \cdot)$  up to  $R$  disappear. It remains

$$\theta_N = \frac{1}{2} \int \int \int w_n \phi_0(b_n, w_n) \phi_0(b_n, w_{\tilde{n}}) db_n dw_n dw_{\tilde{n}}$$

$$+\frac{1}{2} \int \int \int w_{\tilde{n}} \phi_0(b_n, w_n) \phi_0(b_n, w_{\tilde{n}}) db_n dw_n dw_{\tilde{n}} + O(h^{R+1}),$$

since the last two terms involving the  $(R+1)$  derivatives of the Taylor expansion are some  $O(h^{R+1})$ . Thus, by using the Bayes rule,  $\theta_N = \int \int \int w_n \phi_0(w_n|b_n) g_0(b_n) \phi_0(b_n, w_{\tilde{n}}) db_n dw_n dw_{\tilde{n}} = E_0\{E_0[w_n|b_n]g_0(b_n)\} + O(h^{R+1}) = E_0\{w_n g_0(b_n)\} = \theta_0 + O(h^{R+1})$ . Thus  $\sqrt{N}(\theta_N - \theta_0) = O(\sqrt{N}h^{R+1}) = o(1)$  since  $Nh^{2(R+1)} \rightarrow 0$  by A1-(iii).

STEP 4: Thus

$$\begin{aligned} \sqrt{N}(\theta_N - \theta_0) &= \sqrt{N}R_N + \frac{N(N+1)}{N^2} \left\{ \sqrt{N}(U_N - \hat{U}_N) + \sqrt{N}(U_N - \theta_N) + \sqrt{N}(\theta_N - \theta_0) \right\} \\ &\quad - \sqrt{N}\theta_0 + \frac{N(N+1)}{N^2} \sqrt{N}\theta_0. \end{aligned}$$

Using the results of Steps 1–3, we obtained the desired result.  $\square$

## References

- Athey, S.** and **P. Haile** (2002): “Identification of Standard Auction Models,” *Econometrica*, 70, 2107-2140.
- Athey, S.** and **P. Haile** (2005): “Nonparametric Approaches to Auctions,” in J. Heckman and E. Leamer, eds., *Handbook of Econometrics, Volume VI*, Amsterdam: North Holland, forthcoming.
- Athey, S.** and **J. Levin** (2001): “Information and Competition in U.S. Forest Service Timber Auctions,” *Journal of Political Economy*, 109, 375-417.
- Athey, S., J. Levin** and **E. Seira** (2004): “Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions,” Working Paper, Stanford University.
- Bajari, P.** and **A. Hortacsu** (2005): “Are Structural Estimates of Auction Models Reasonable? Evidence from Experimental Data,” *Journal of Political Economy*, 113, 703-741.
- Baldwin, L.** (1995): “Risk Averse Bidders at Oral Ascending-Bid Forest Service Timber Sales,” Working Paper, RAND Corporation.
- Baldwin, L., R. Marshall** and **J.F. Richard** (1997): “Bidder Collusion at Forest Service Timber Sales,” *Journal of Political Economy*, 105, 657-699.
- Bhattacharya, P.** and **A. Gangopadhyay** (1990): “Kernel and Nearest-Neighbor Estimation of a Conditional Quantile,” *Annals of Statistics*, 18, 1400-1415.
- Brendstrup, B.** and **H. Paarsch** (2006): “Identification and Estimation in Sequential Asymmetric English Auctions,” *Journal of Econometrics*, forthcoming.
- Campo, S.** (2005): “Attitudes Toward Risk and Asymmetric Bidding: Evidence from Construction Procurements,” Working Paper, University of North Carolina.
- Campo, S., E. Guerre, I. Perrigne** and **Q. Vuong** (2006): “Semiparametric Estimation of First-Price Auctions with Risk Averse Bidders,” Working Paper, Pennsylvania State University.

- Campo, S., I. Perrigne and Q. Vuong** (2003): “Asymmetry in First-Price Auctions with Affiliated Private Values,” *Journal of Applied Econometrics*, 18, 197-207.
- Chaudhuri, P.** (1991): “Global Nonparametric Estimation of Conditional Quantile Functions and their Derivatives,” *Journal of Multivariate Analysis*, 39, 246-269.
- Cox, J., V. Smith and J. Walker** (1988): “Theory and Individual Behavior of First-Price Auctions,” *Journal of Risk and Uncertainty*, 1, 61-99.
- Eso, P. and L. White** (2004): “Precautionary Bidding in Auctions,” *Econometrica*, 72, 77-92.
- Flambard, V. and I. Perrigne** (2006): “Asymmetry in Procurement Auctions: Some Evidence from Snow Removal Contracts,” *The Economic Journal*, 116, 1-23.
- Goere, J., C. Holt and T. Palfrey** (2002): “Quantal response Equilibrium and Overbidding in Private Value Auctions,” *Journal of Economic Theory*, 104, 247-272.
- Gollier, C.** (2001): *The Economics of Risk and Time*, MIT Press.
- Guerre, E., I. Perrigne and Q. Vuong** (2000): “Optimal Nonparametric Estimation of First-Price Auctions,” *Econometrica*, 68, 525-574.
- Guerre, E., I. Perrigne and Q. Vuong** (2006): “Nonparametric Identification of Risk Aversion in First-Price Auctions Under Exclusion Restrictions,” Working Paper, Pennsylvania State University.
- Haile, P.** (2001): “Auctions with Resale Markets: An Application to US Forest Service Timber Sales,” *American Economic Review*, 91, 399-427.
- Hansen, R.** (1985): “Empirical Testing of Auction Theory,” *American Economic Review*, 75, 156-159.
- Maskin, E. and J. Riley** (1984): “Optimal Auctions with Risk Averse Buyers,” *Econometrica*, 52, 1473-1518.

- Matthews, S.** (1987): “Comparing Auctions for Risk Averse Buyers: A Buyer’s Point of View,” *Econometrica*, 633-646.
- Perrigne, I.** (2003): “Random Reserve Prices and Risk Aversion in Timber Auctions,” Working Paper, Pennsylvania State University.
- Pinkse, J.** and **G. Tan** (2005): “The Affiliation Effect in First-Price Auctions,” *Econometrica*, 73, 263-277.
- Powell, J.L.** (1994): “Estimation of Semiparametric Models,” in R.F. Engle and D.L. McFadden, eds., *Handbook of Econometrics, Volume IV*, Amsterdam: North Holland.
- Powell, J.L., J. H. Stock** and **T.M. Stoker** (1989): “Semiparametric Estimation of Index Coefficients,” *Econometrica*, 57, 1403-1430.
- Pratt, J.** (1964): “Risk Aversion in the Small and in the Large,” *Econometrica*, 32, 122-136.
- Serfling, R.J.** (1980): *Approximation Theorems of Mathematical Statistics*, New York: John Wiley.
- Stone, C.J.** (1982): “Optimal Rates of Convergence for Nonparametric Regressions,” *The Annals of Statistics*, 10, 1040-1053.
- Vuong, Q.** (2003): “ $\sqrt{N}$  Estimation of the Average Density,” lecture notes.

Figure 1:  $(volume, appraisal\ value), I = 2$

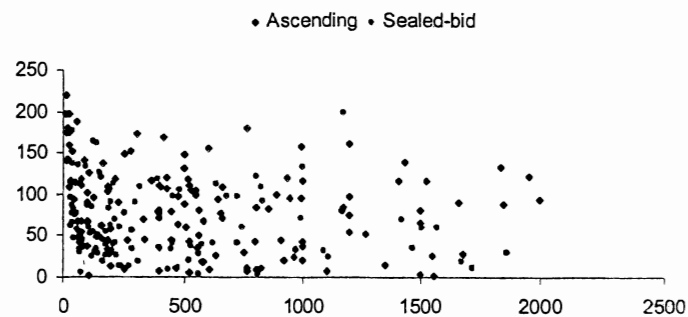


Figure 2:  $(volume, appraisal\ value), I = 3$

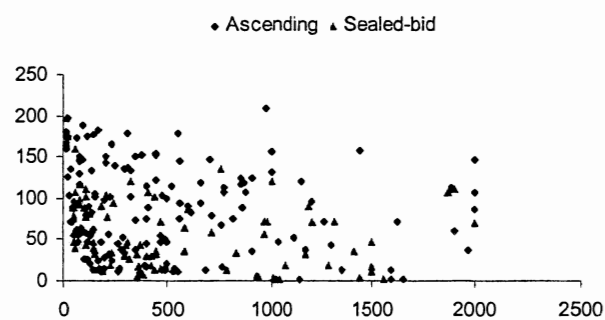


Figure 3: Conditional Private Value Distributions

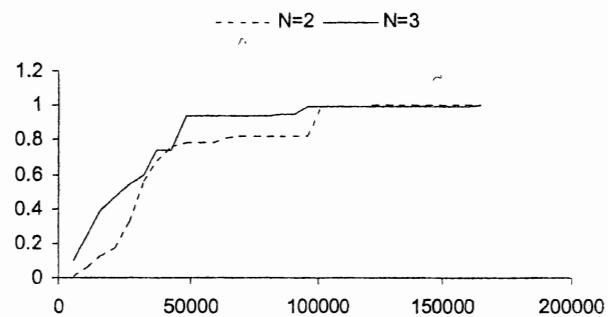


Figure 4: Conditional Private Value Densities

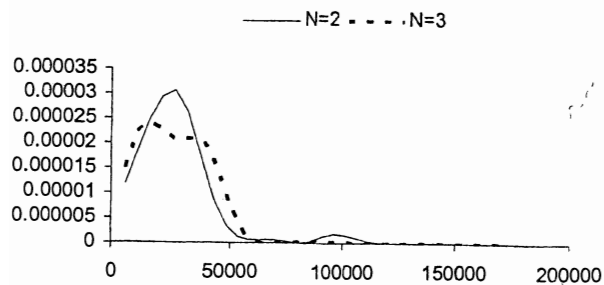




Figure 5: Conditional Bid and Private Value Distributions,  $I = 2$

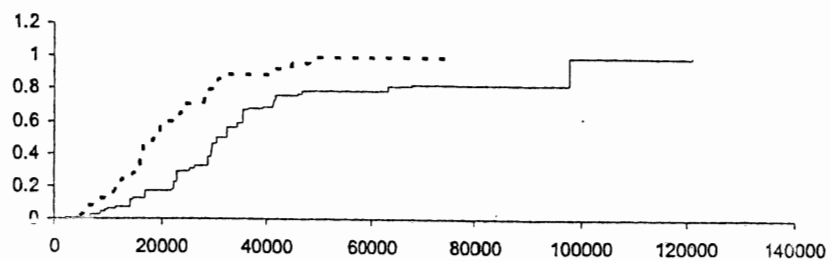


Figure 6: Conditional Bid and Private Value Distributions,  $I = 3$

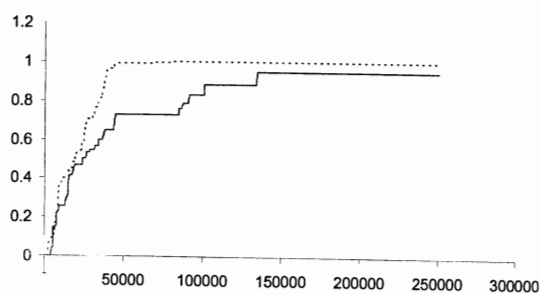


Figure 7:  $\lambda(\cdot)$ ,  $I = 2$

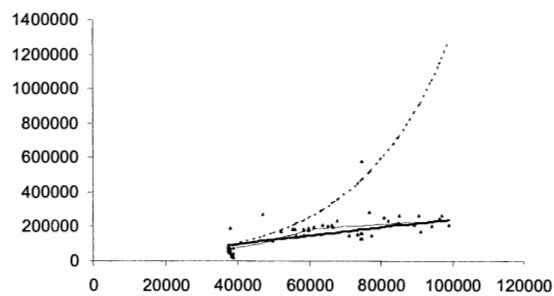


Figure 8:  $\lambda(\cdot)$ ,  $I = 3$

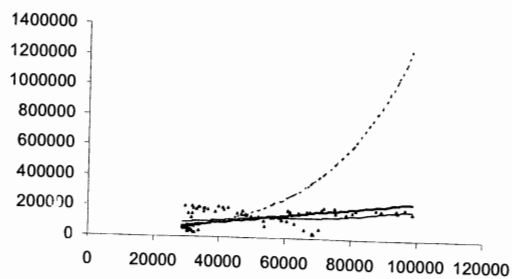


Figure 9:  $U(\cdot)$ ,  $I = 2$

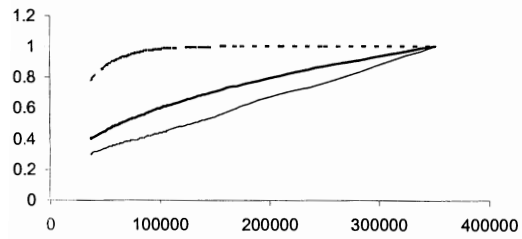


Figure 10:  $U(\cdot)$ ,  $I = 3$

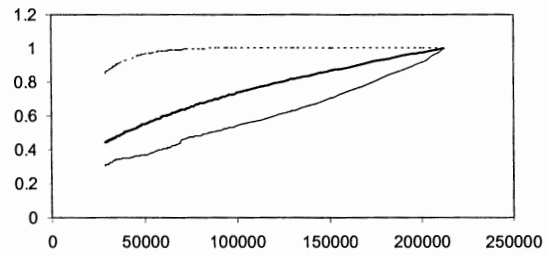


Figure 11: Private Value Distributions,  $I = 2$

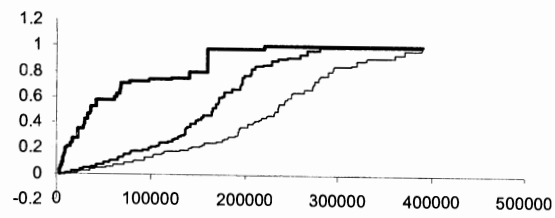


Figure 12: Private Value Distributions,  $I = 3$

